

S38-7

# Integral-type Time-to-Digital Converter

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JAPAN

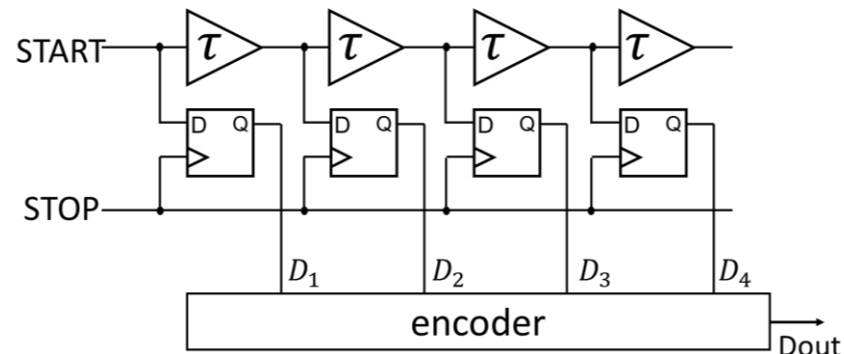


Kobayashi Lab.  
Gunma University

# Motivation

## TDC Architectures with NO delay lines

- for higher time resolution
- to avoid PVT variations of delay lines:
  - Process
  - Voltage
  - Temperature



Conventional TDC

# Outline

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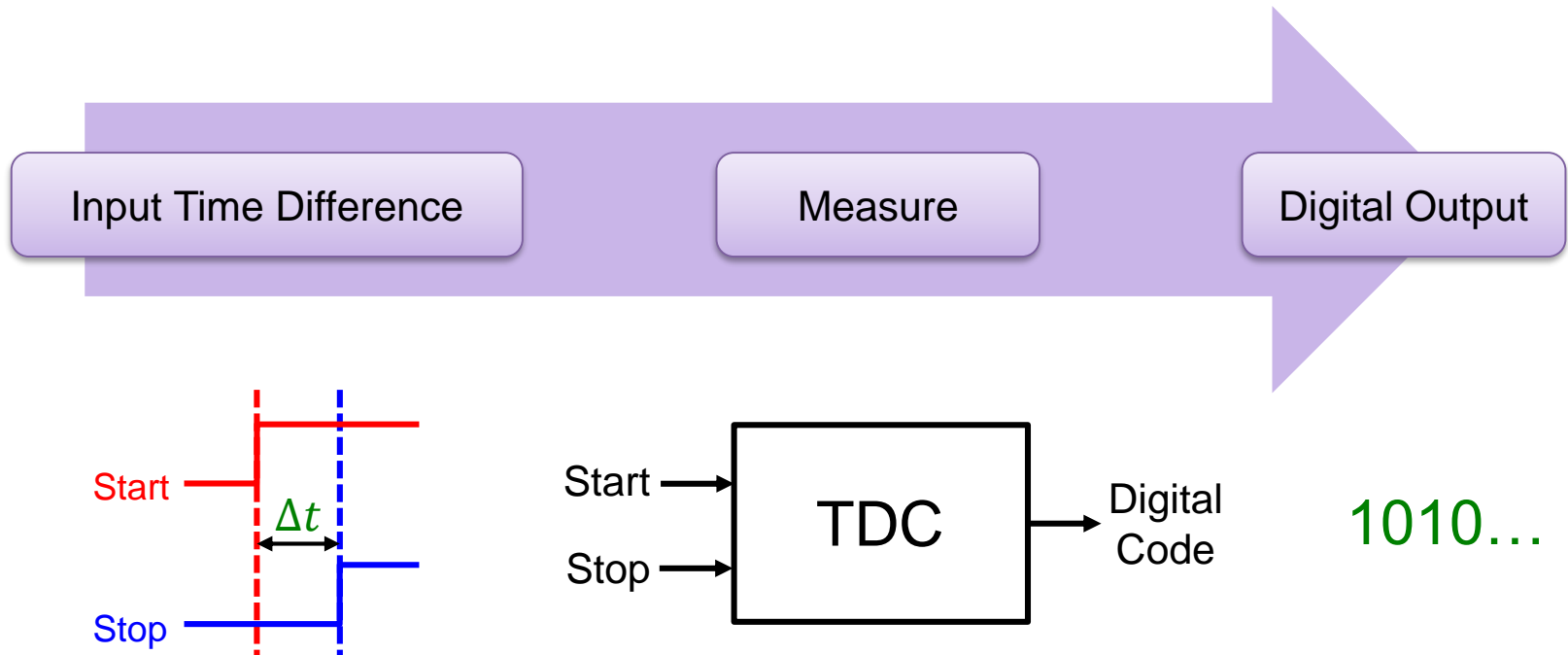
- Introduction
- Proposed TDC Architecture and Operation
- Highly Efficient Data Acquisition Condition
- Jitter Effects
- Summary

# Outline

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# Time-to-Digital Converter

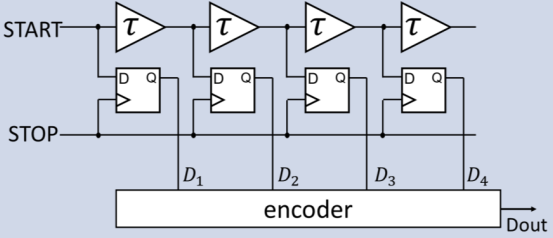
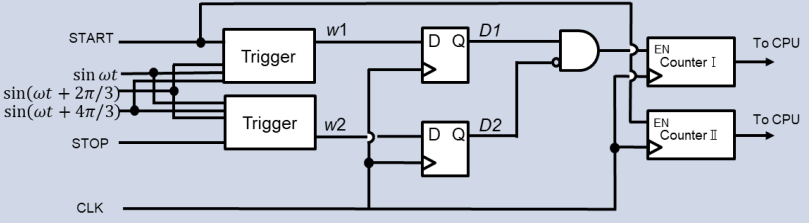


## Time-to-Digital Converter (TDC) :

measures timing difference between two input signals as a digital code



# Comparison

	Conventional TDC	Proposed TDC
Architecture		
Delay Line	YES	NO
PVT Variation	Large	Small
Self Calibration	Required	Not Required
Time Resolution	Low	High
Measuring Time	Short	Long
Analog Circuit	NO	YES

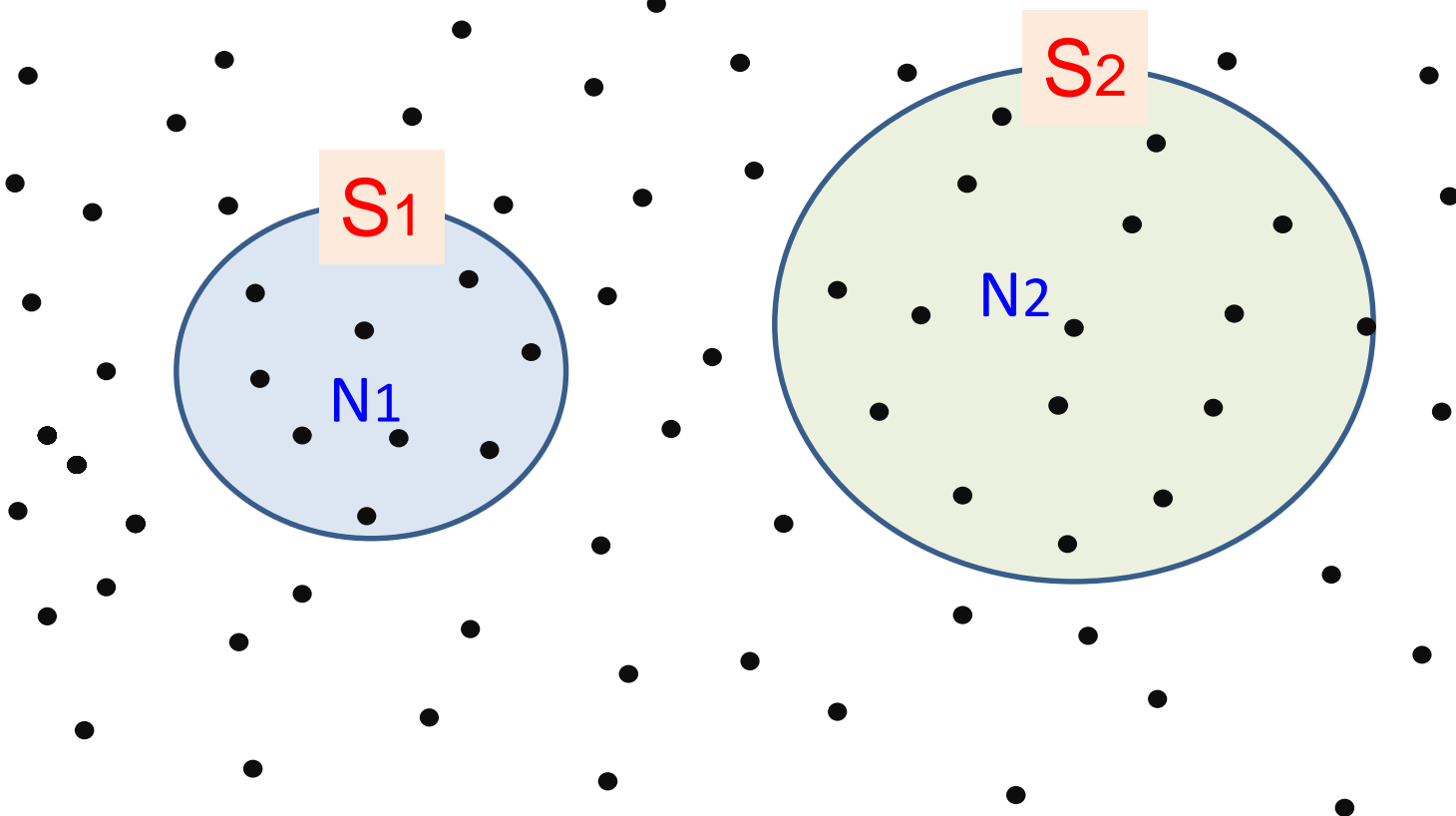
# Outline

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# Probabilistic Measurement

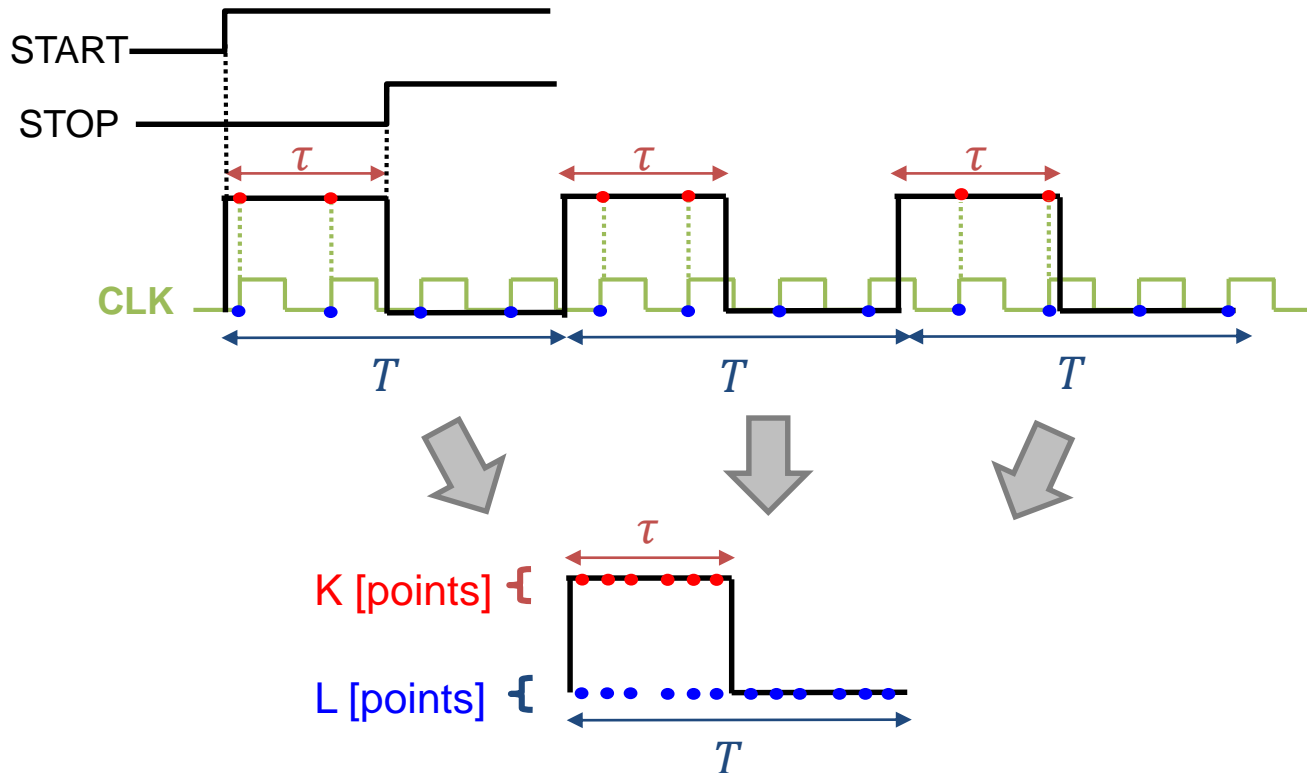
Random dots (Monte Carlo Method)



# of dots ratio  $\frac{N_1}{N_2}$   $\longrightarrow$  Area ratio  $\frac{S_1}{S_2}$



# Proposed TDC Principle (1/3)

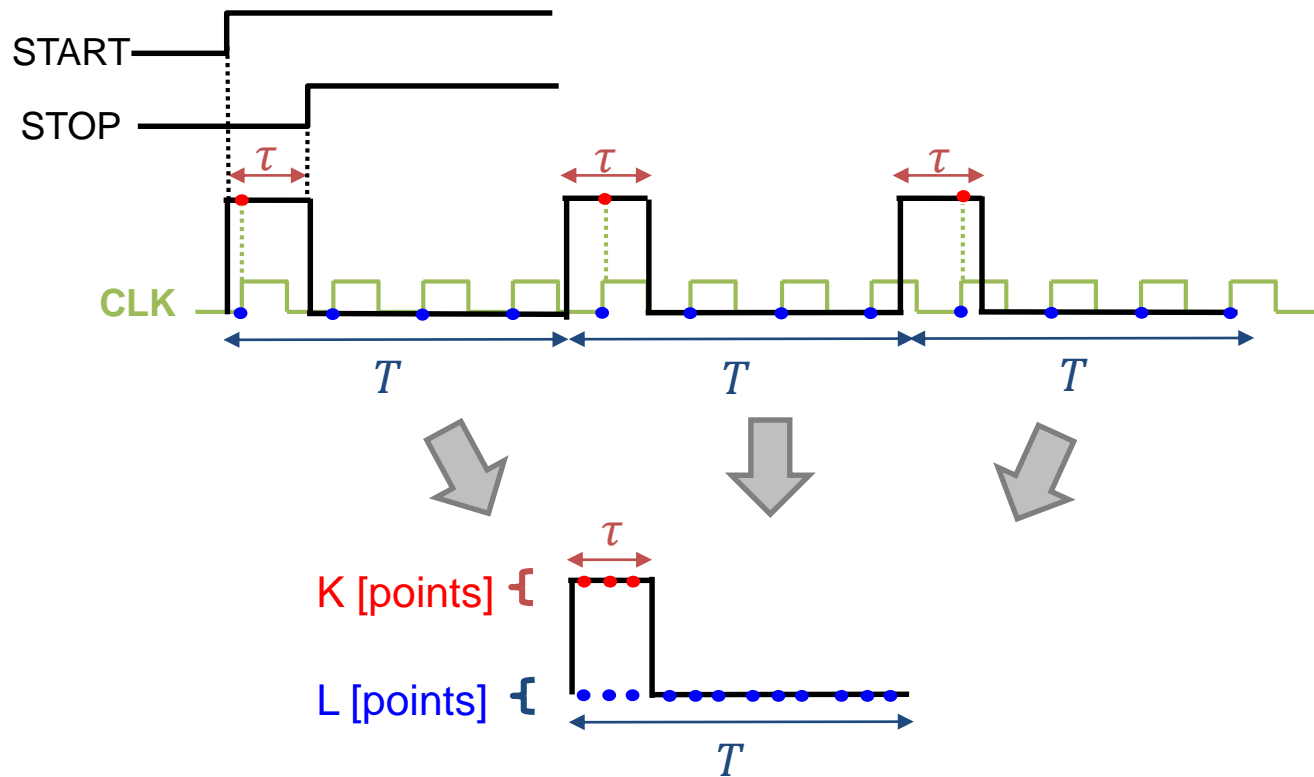


Sampling a square wave with **input time difference  $\tau$**  / **reference period  $T$**  duty cycle



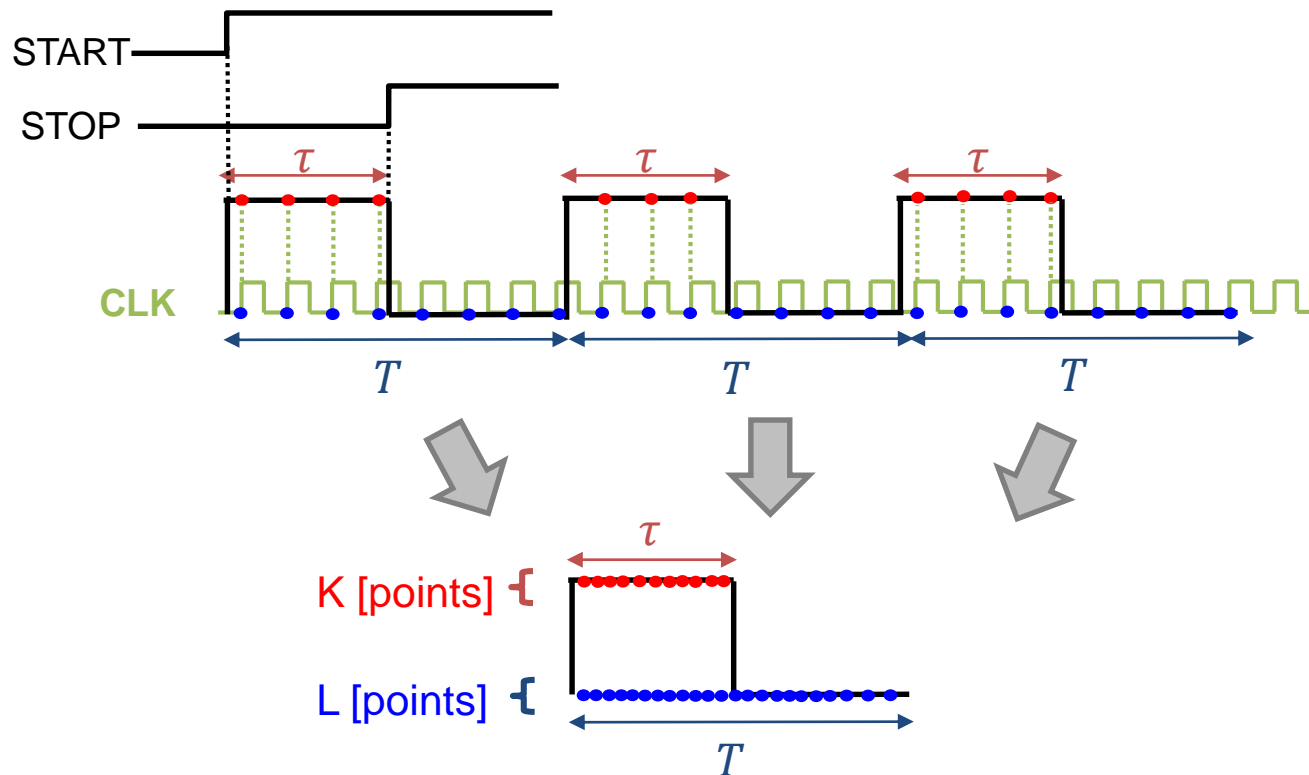
$$\lim_{L \rightarrow \infty} \frac{K}{L} = \frac{\tau}{T}$$

# Proposed TDC Principle (2/3)



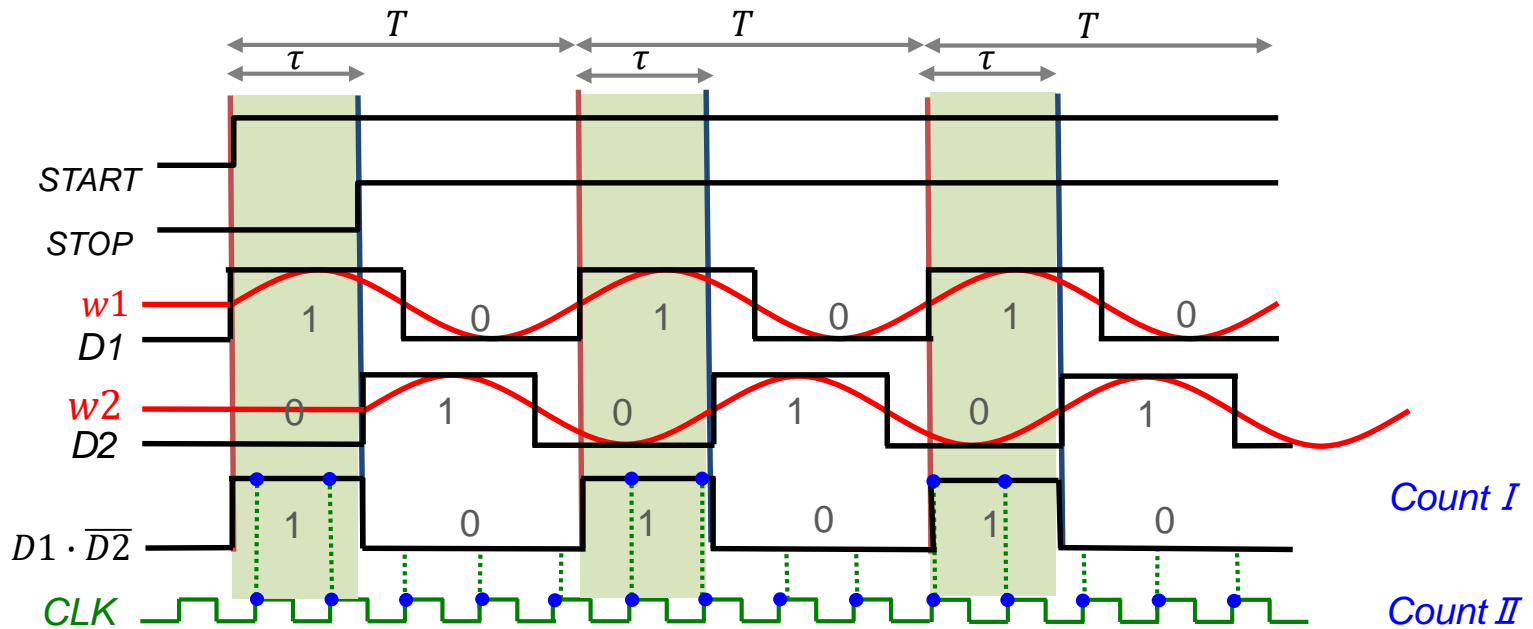
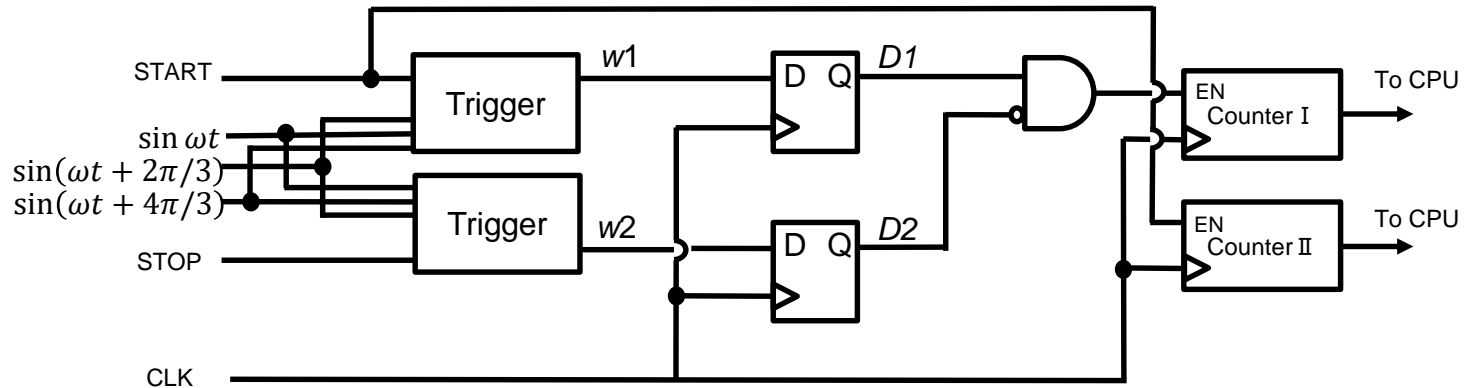
Square wave duty cycle depends on input time difference  $\tau$

# Proposed TDC Principle (3/3)

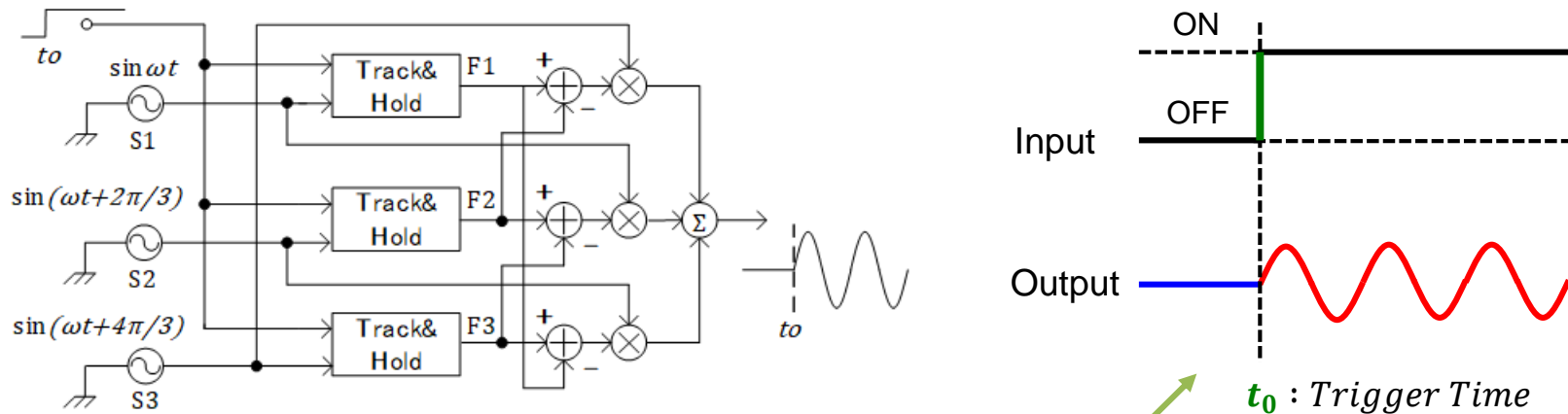


Acquiring more data improves time resolution

# Proposed TDC Architecture



# Oscilloscope-Trigger Circuit (1/2)

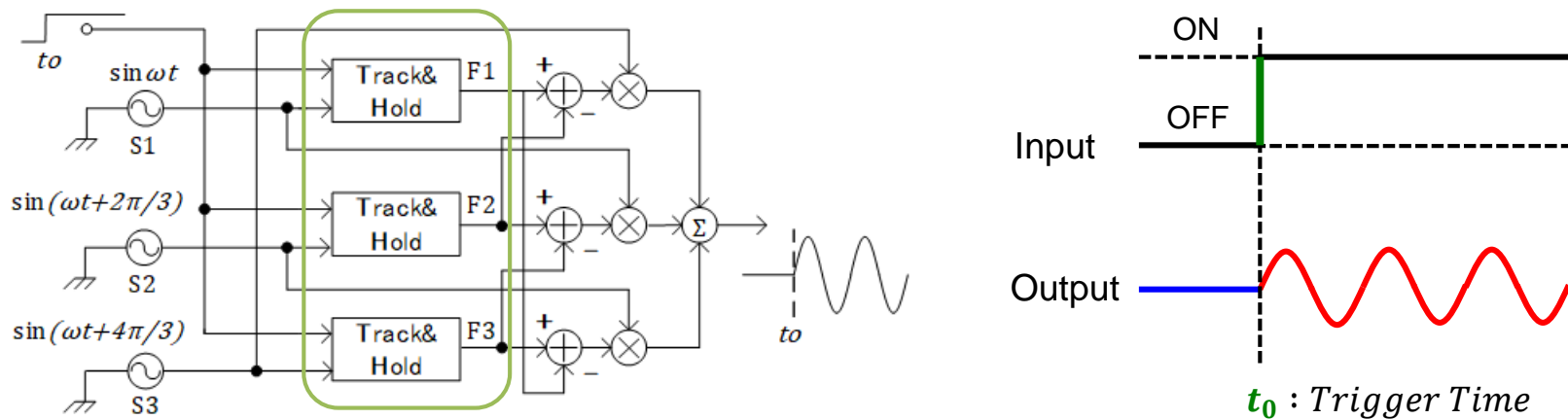


Output starts to oscillate at **rising edge timing of input** from **phase 0**

[1] M. Nelson (Tektronics)

"A New Technique for Low-Jitter Measurements Using Equivalent-Time Sampling Oscilloscope"  
Automatic RF Techniques Group 56th Measurement (Dec. 2000)

# Oscilloscope-Trigger Circuit (2/2)



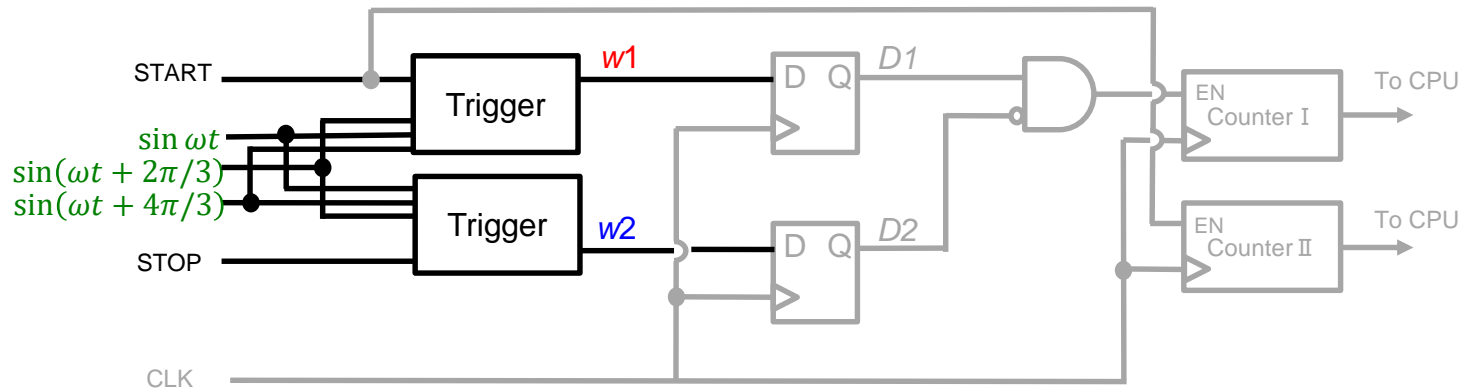
## Track mode:

$$\begin{aligned}
 V_{out} = & \sin(\omega t + 4\pi/3) \{ \sin \omega t - \sin(\omega t + 2\pi/3) \\
 & + \sin \omega t \{ \sin(\omega t + 2\pi/3) + \sin(\omega t + 4\pi/3) \} \\
 & + \sin(\omega t + 2\pi/3) \{ \sin(\omega t + 2\pi/3) + \sin(\omega t + 4\pi/3) \} = \mathbf{0}
 \end{aligned}$$

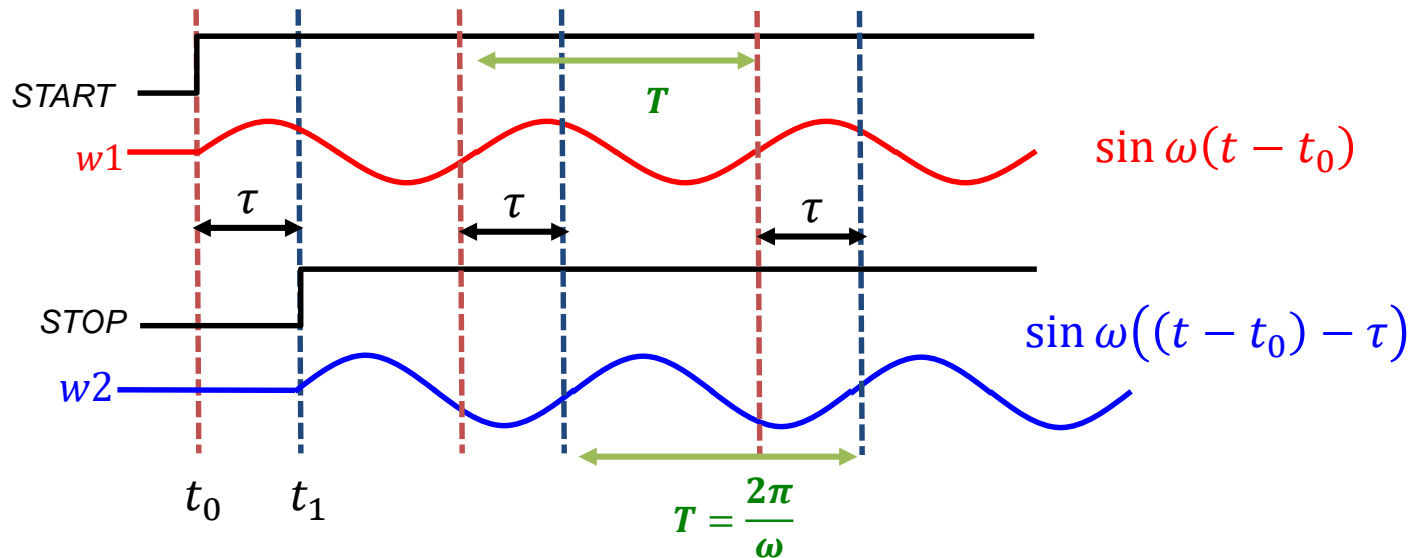
## Hold mode:

$$\begin{aligned}
 V_{out} = & \sin(\omega t + 4\pi/3) \{ \sin \omega t_0 - \sin(\omega t_0 + 2\pi/3) \\
 & + \sin \omega t \{ \sin(\omega t_0 + 2\pi/3) + \sin(\omega t_0 + 4\pi/3) \} \\
 & + \sin(\omega t + 2\pi/3) \{ \sin(\omega t_0 + 2\pi/3) + \sin(\omega t_0 + 4\pi/3) \} \\
 = & \mathbf{((3\sqrt{3})/2)\sin(\omega(t - t_0))}
 \end{aligned}$$

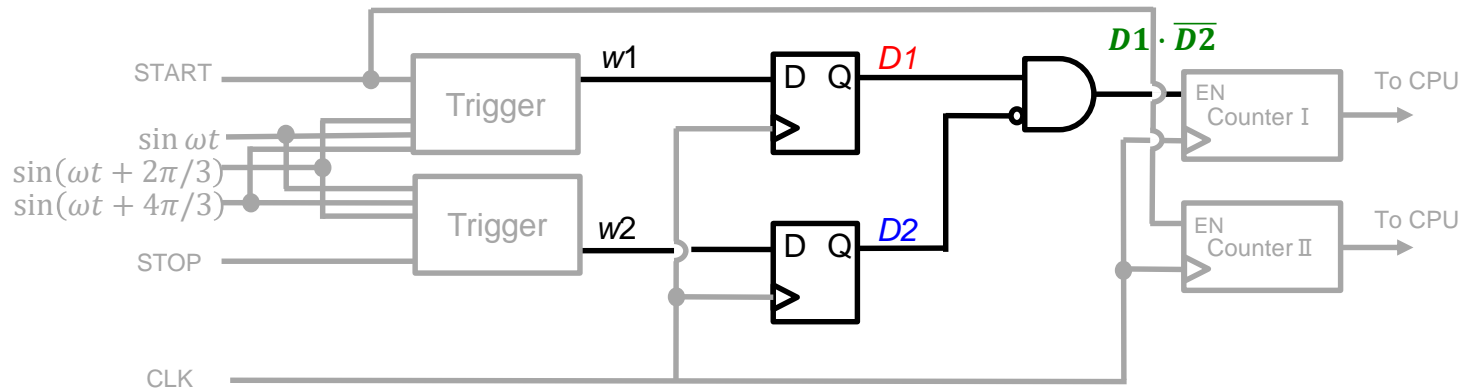
# Proposed TDC Operation (1/3)



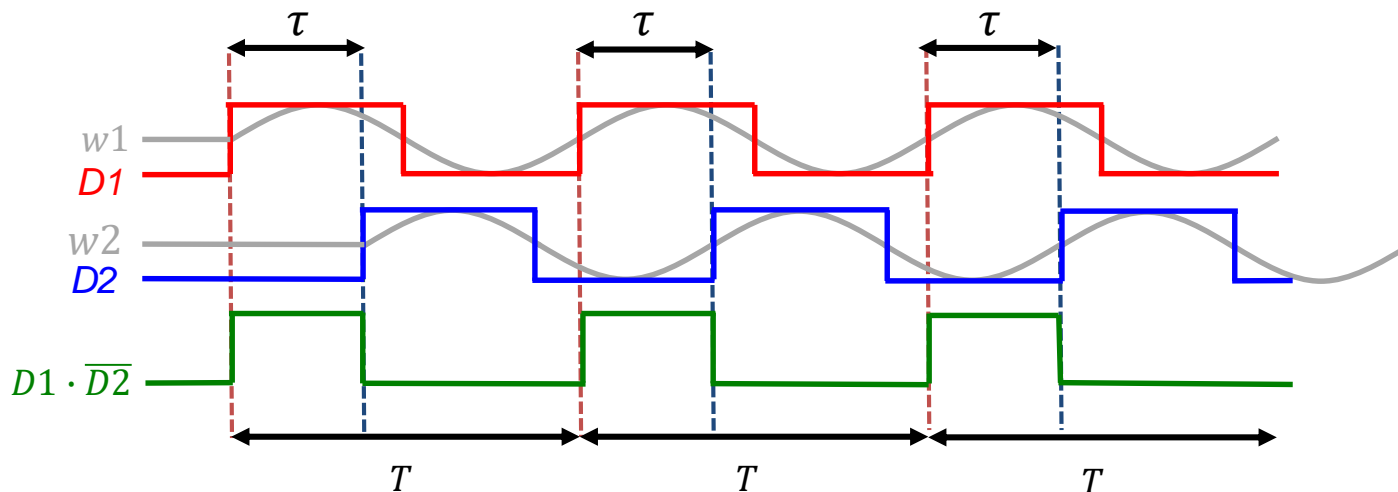
STEP1: Holding the input time difference  $\tau$  as phase difference



# Proposed TDC Operation (2/3)

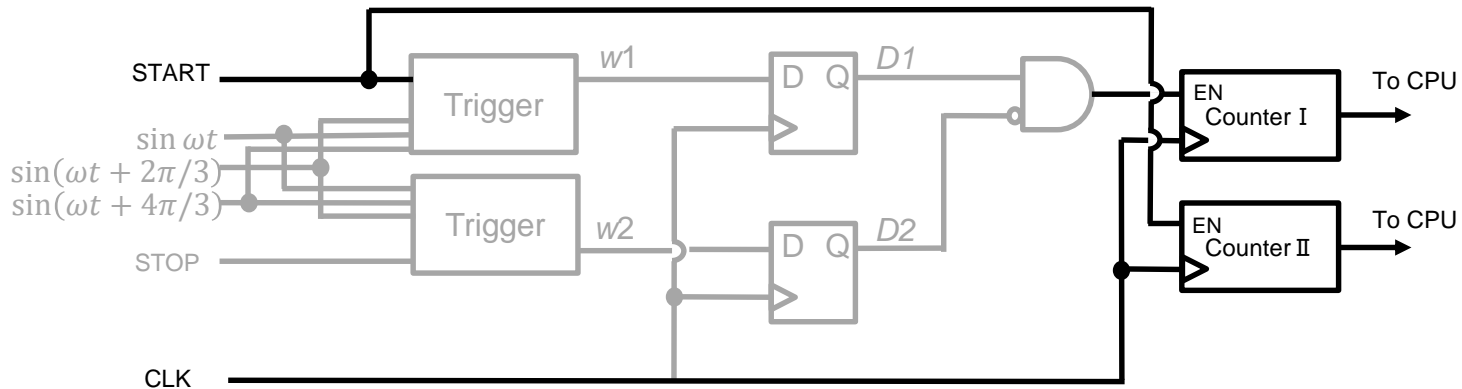


STEP2: Making the square wave with  $\tau / T$  duty cycle

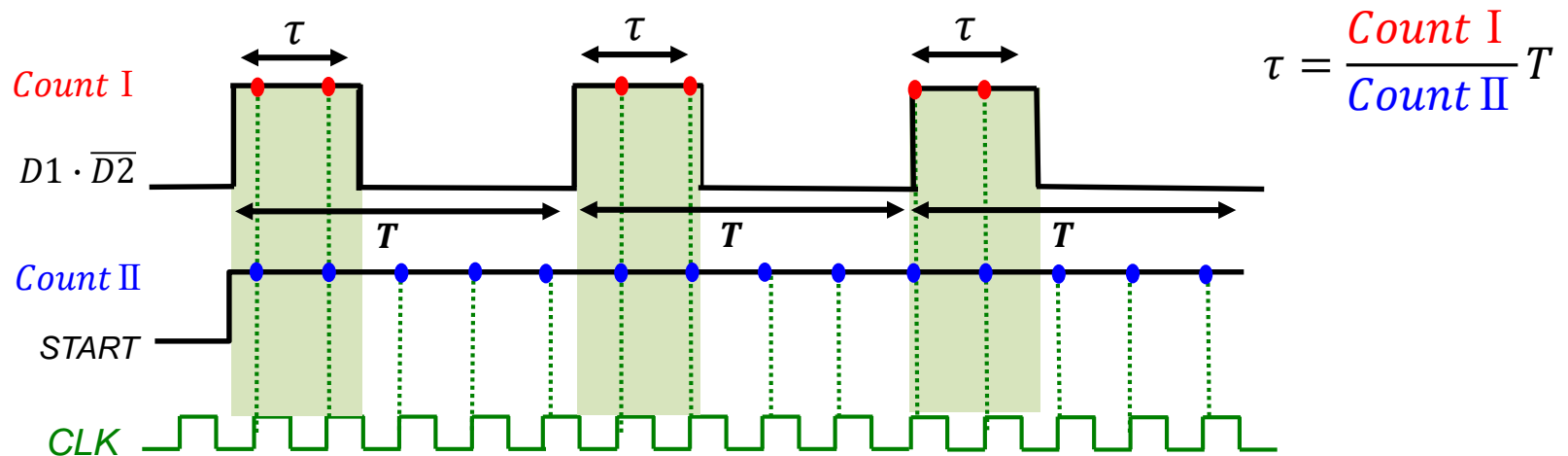




# Proposed TDC Operation (3/3)



STEP3: Counting the ratio of the sampling points

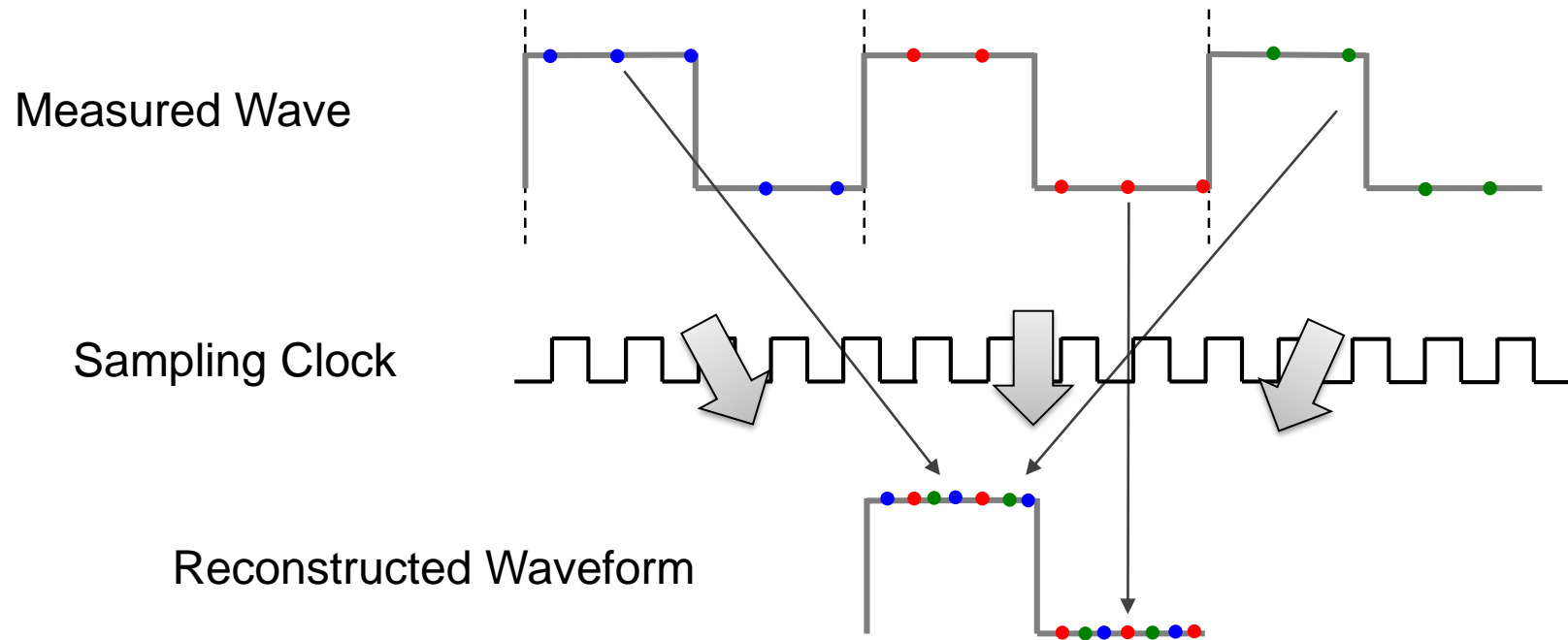


# Outline

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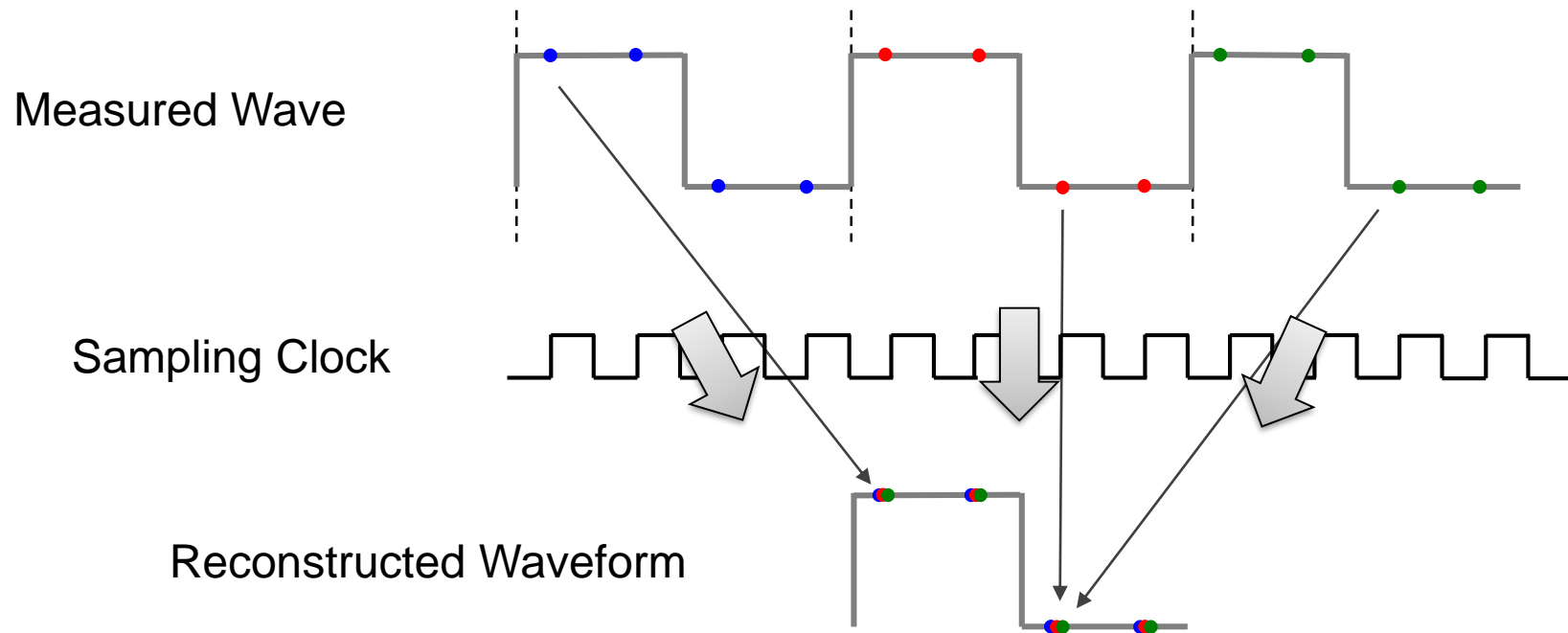
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# Equivalent-Time Sampling



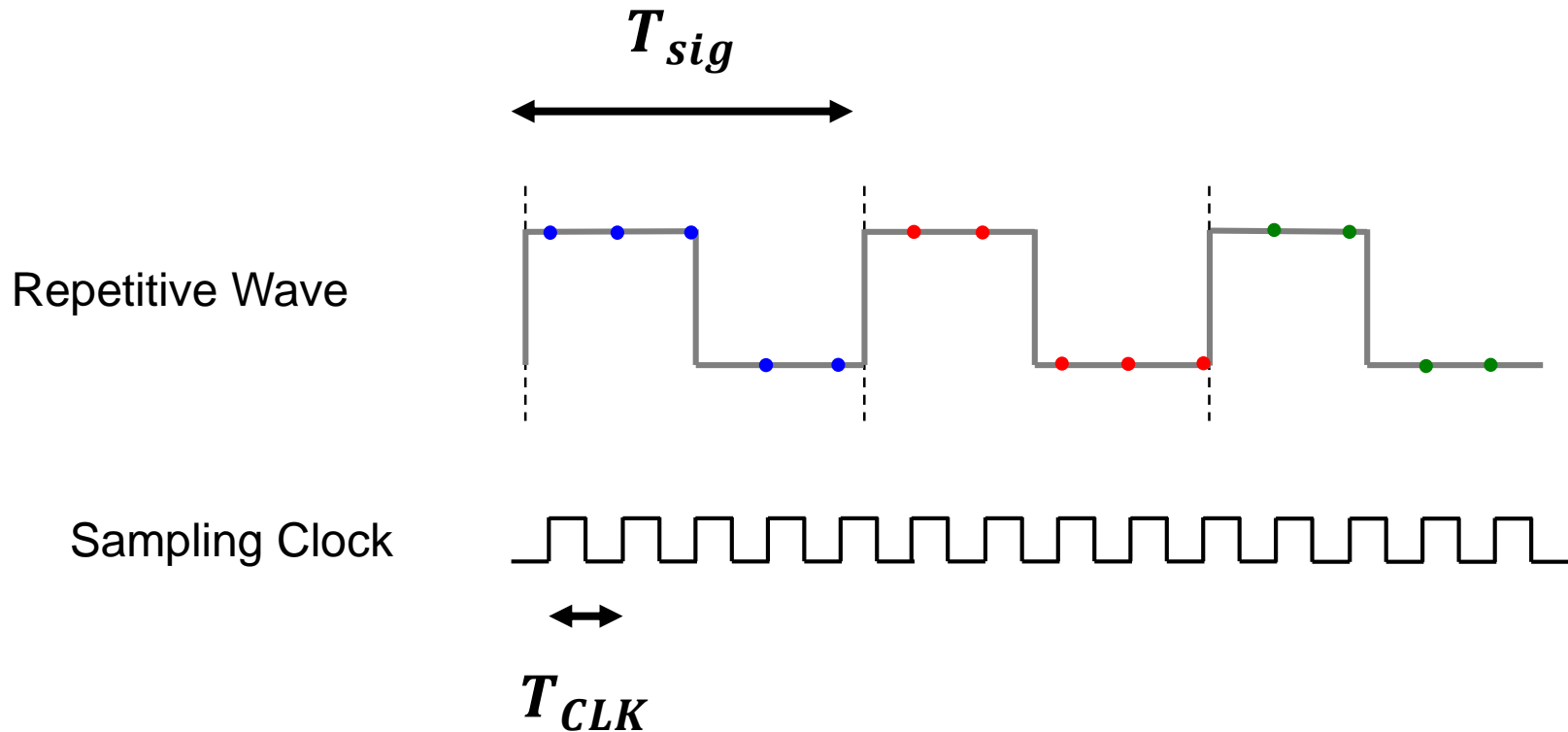
Higher time resolution than sampling clock period

# Waveform Missing



Sampling points must be dispersed uniformly

# Data Acquisition Condition

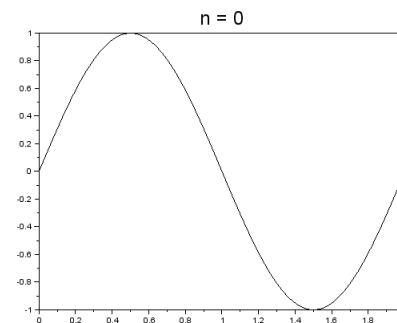
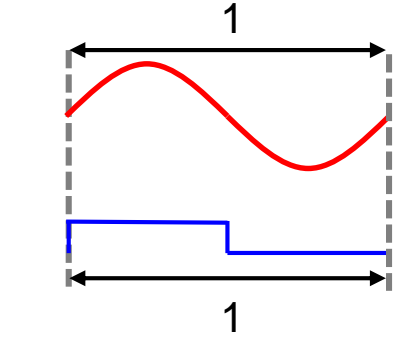
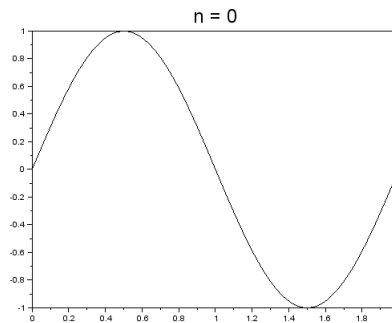
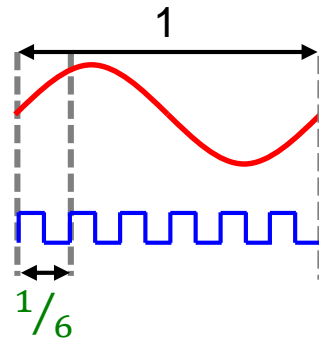
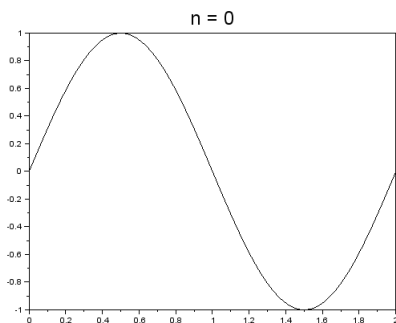
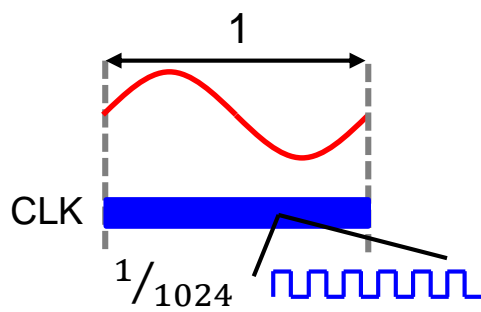


$$T_{CLK} = \boxed{?} \times T_{sig}$$

# Waveform Missing Condition

$$f_{CLK} \gg f_{sin} \quad f_{CLK} \approx \frac{1}{\alpha} f_{sin} \quad \left( \alpha = 1, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \dots, \frac{1}{6}, \dots \right)$$

$$f_{CLK} \approx f_{sin}$$

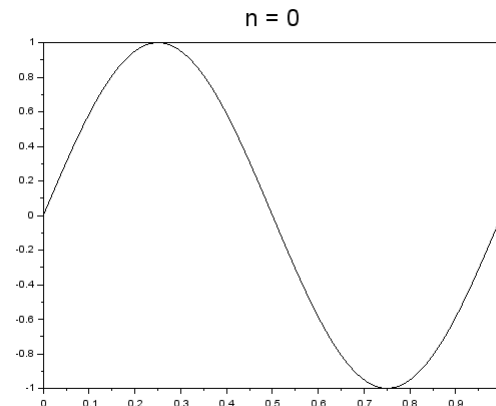
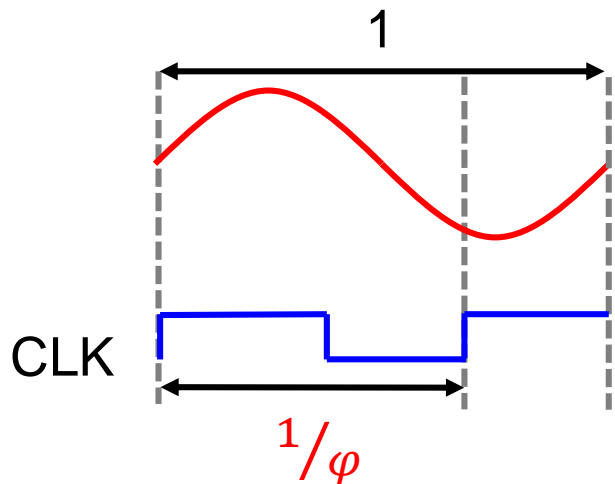


Sampling points move little  $\rightarrow$  Requires long time

# Highly Efficient Condition

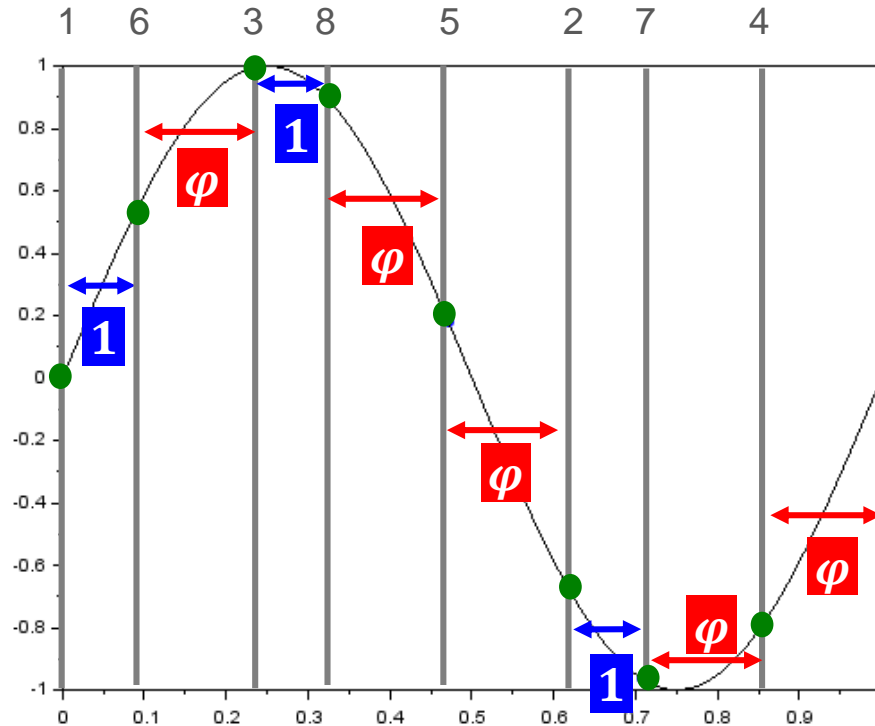
$$f_{CLK} = \varphi \times f_{sig}$$

$\varphi$  : Golden ratio ( = 1.6180339887... )



Sampling points are dispersed uniformly through measurement

# Golden Ratio Sampling



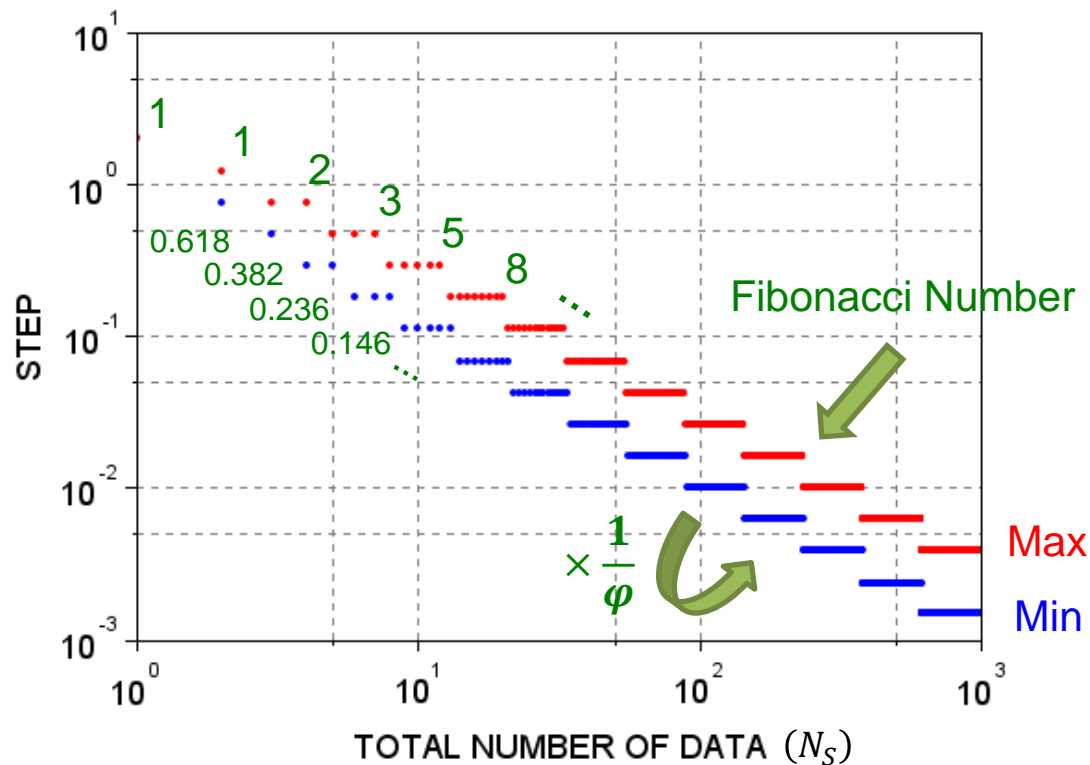
All sections are divided by golden ratio

➔ **Max / Min distances =  $\varphi$  or  $\varphi^2$  const.**



# Time Resolution

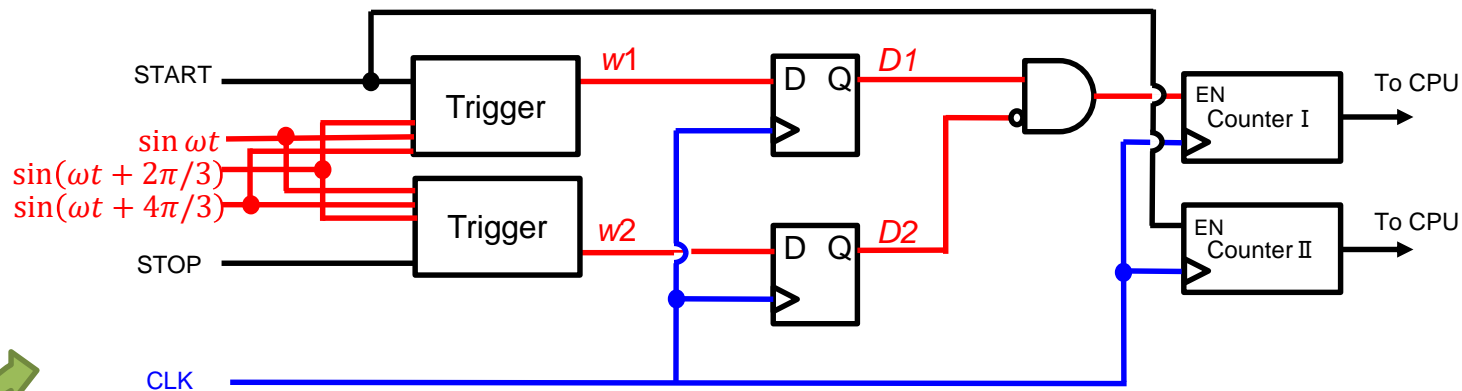
**Max** & **Min** distances between neighbor points vs. Total Number of Data



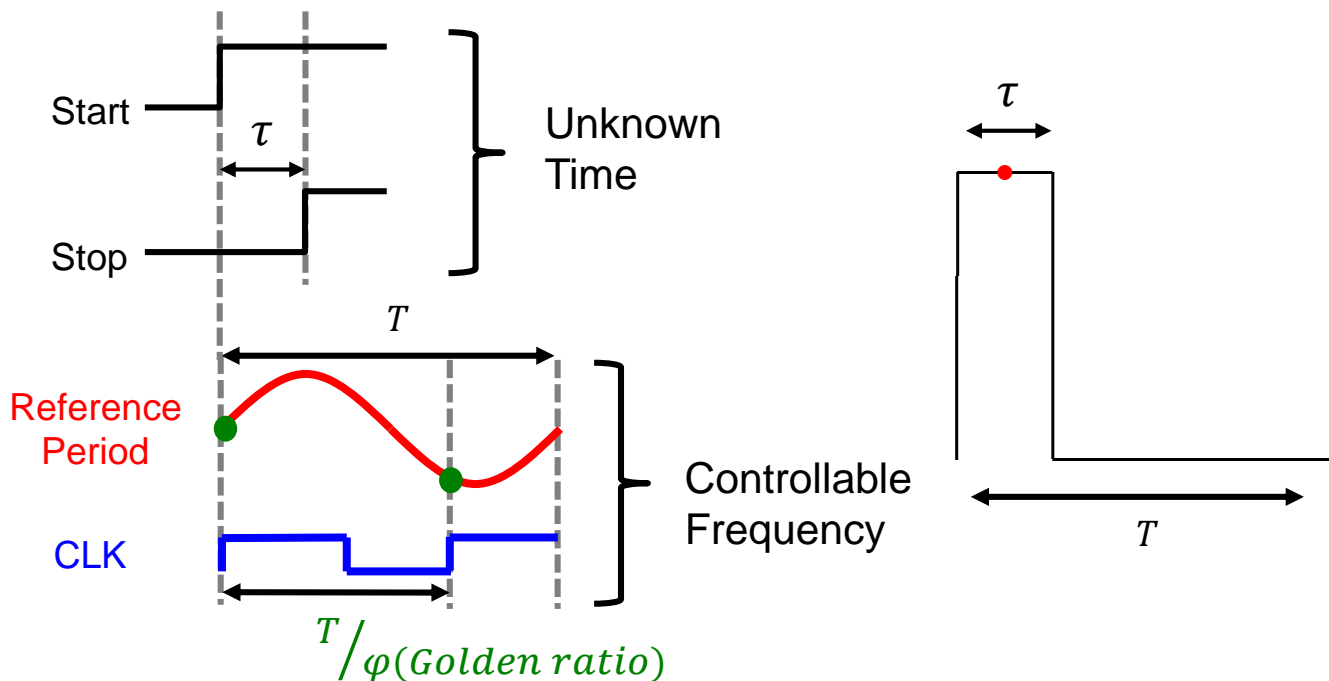
**Max** & **Min** distances decreases  $\times 1/\phi$  every **Fibonacci numbers**

**➔ Time resolution improves about  $1 / \text{Total Number of data}$**

# Proposed TDC Data Acquisition

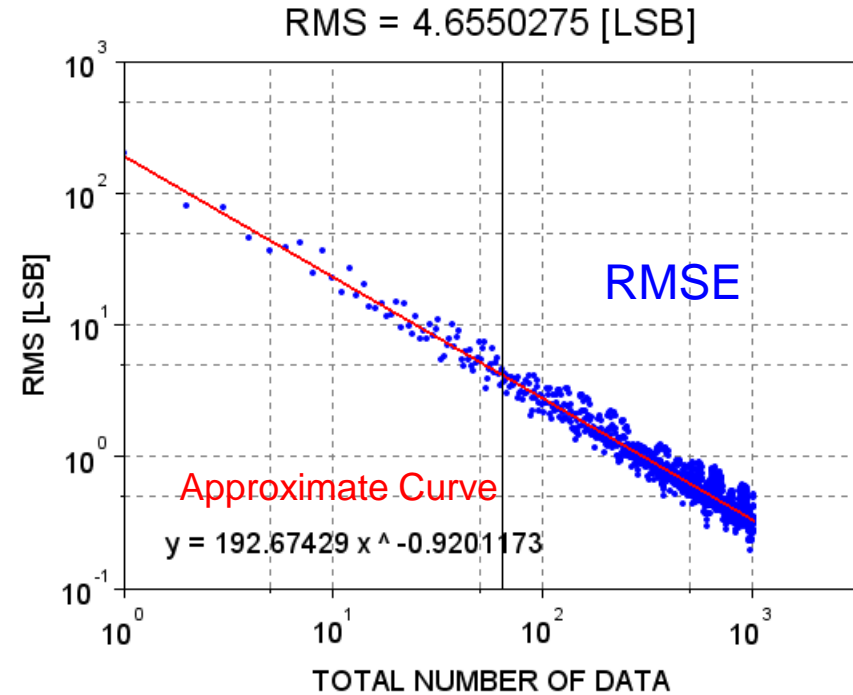
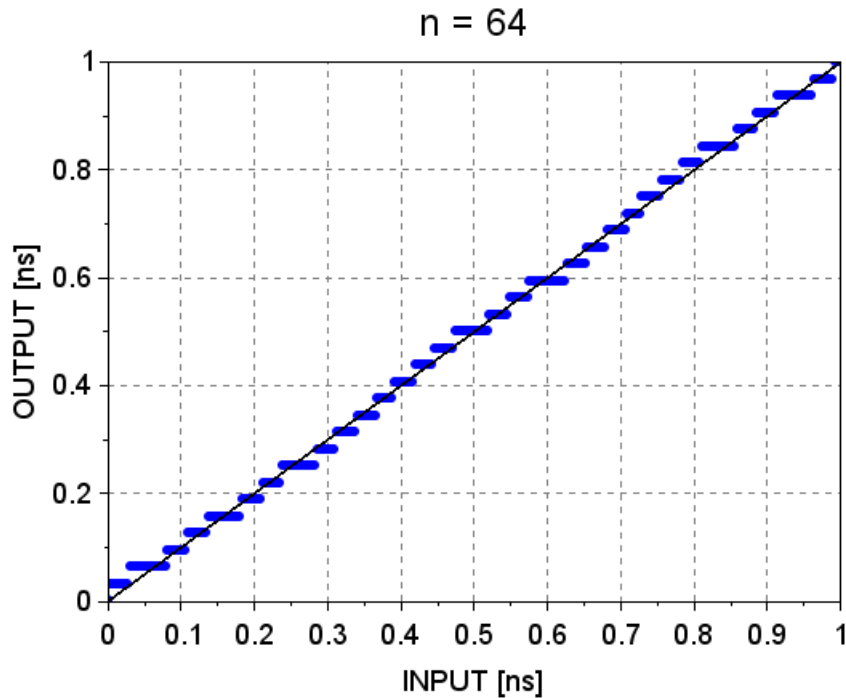


input



output

# Simulation Result



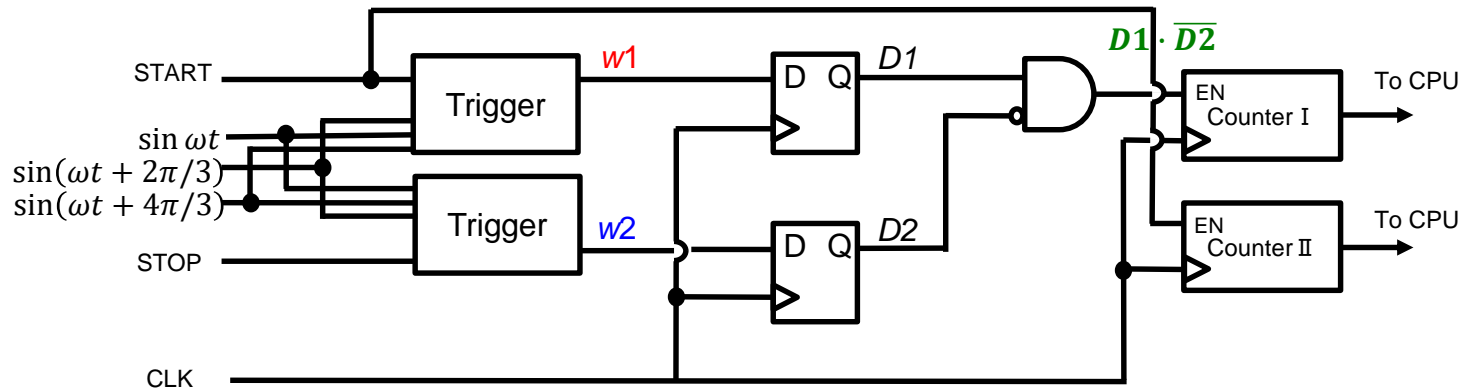
Acquiring more data improves time resolution

# Outline

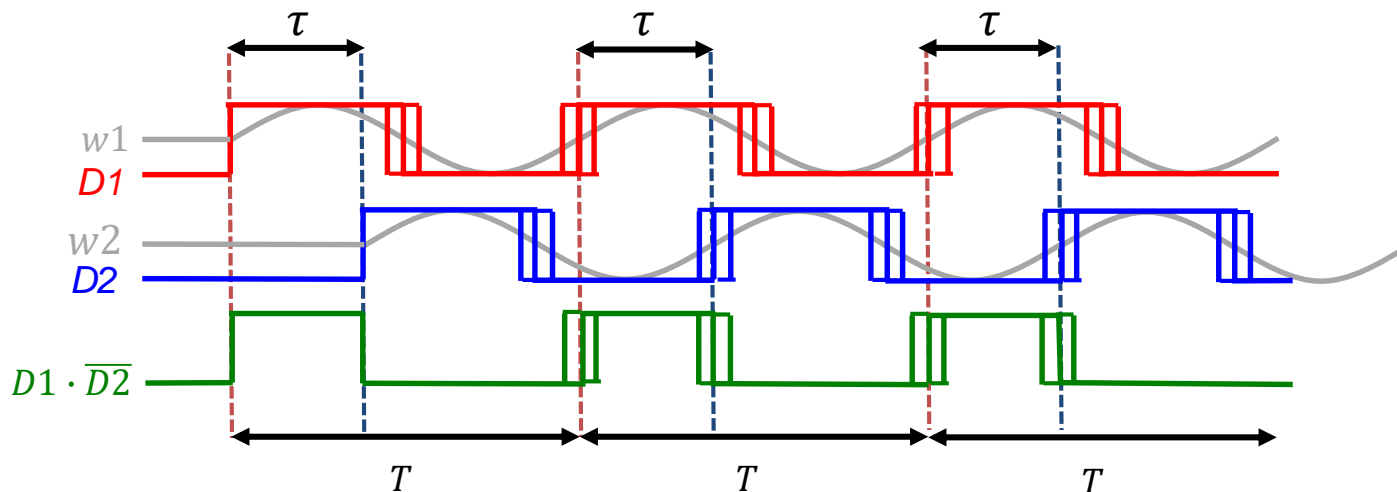
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# Jitter Effects

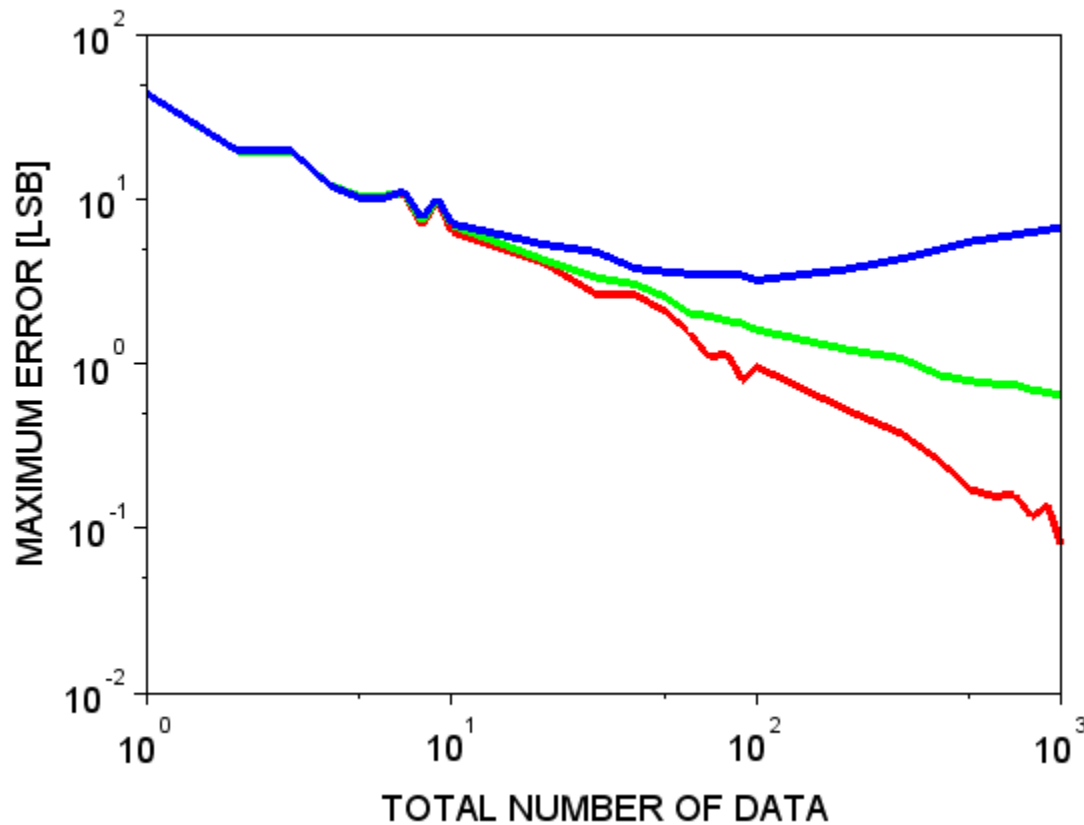


Jitter of  $w1$  &  $w2$  affects period & duty of  $D1 \cdot \overline{D2}$



# Simulation Result (1/2)

## Maximum Error vs. Total Number of Data



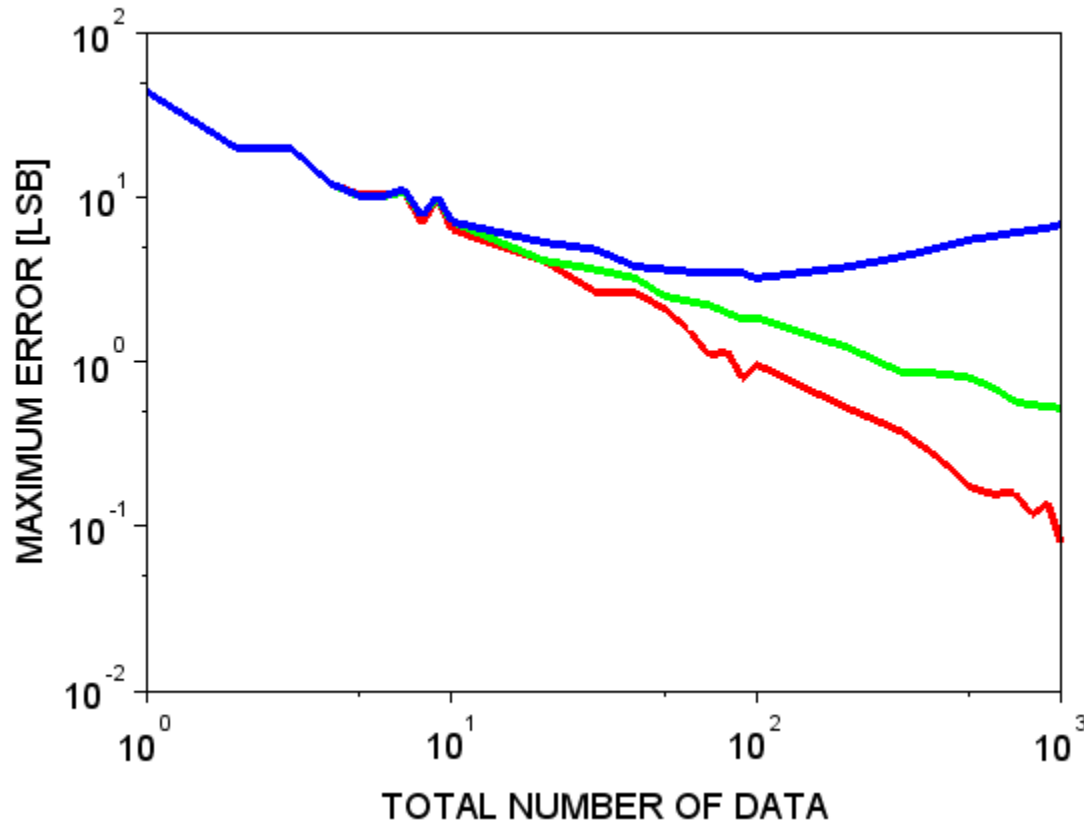
— No Jitter

— Random Jitter

— Accumulated Jitter

# Simulation Result (2/2)

## Maximum Error vs. Total Number of Data



— No Jitter

— Common  
Accumulated Jitter

— Differential  
Accumulated Jitter

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# Summary

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- Proposed integral-type TDC:
  - fine time resolution
  - no need for calibration
- Highly Efficient Data Acquisition Condition:
  - Sampling clock frequency / measured signal frequency  
= Golden ratio
- Robust for jitter

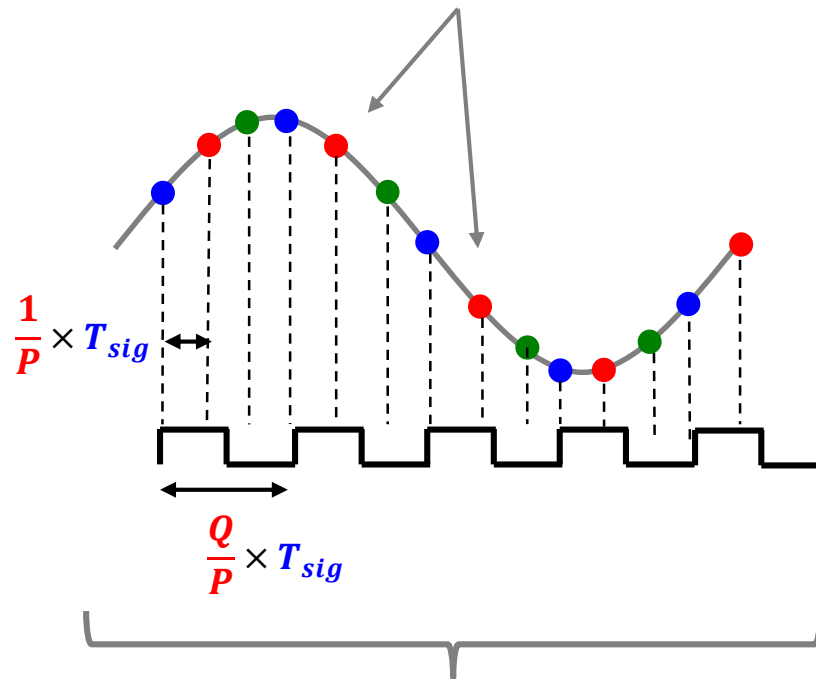
# Appendix

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# Deterministic Measurement

$$T_{CLK} = \frac{Q}{P} \times T_{sig} \quad ( P, Q: \text{integers and relatively prime} )$$

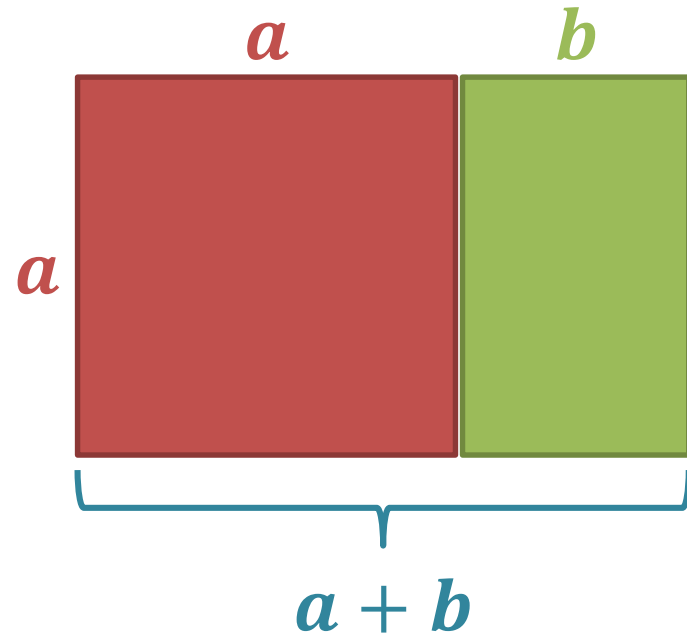
**Q:** Phase distance for each sampling



**P:** Maximum number of total measurable sampling points

# Golden Ratio

$$\varphi \equiv \frac{a+b}{a} = \frac{a}{b}$$



$$\varphi = \frac{1 + \sqrt{5}}{2} = 1.6180339887 \dots$$

# Fibonacci Number

$$F_0 = 0$$

$$F_1 = 1$$

$$F_{n+2} = F_n + F_{n+1}$$



0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

$$\lim_{n \rightarrow \infty} \frac{F_n}{F_{n-1}} = 1.6180339887 \dots = \varphi$$